RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [BATCH 2014-17]
B.A./B.Sc. FOURTH SEMESTER (January – June) 2016
Mid-Semester Examination, March 2016

Date : 17/03/2016 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : IV Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

1. **Answer any five questions :** [5×3]

a) Suppose $\mathcal{P}(\mathbb{N})$ denotes the power set of $\mathbb{N}$ and consider the metric 'd'on $\mathcal{P}(\mathbb{N})$ defined by
$$d(A,B)=\begin{cases} 0, & A\Delta B=\phi \\ \frac{1}{m}, & m \text{ is the least element of } A\Delta B \end{cases}$$
Find diameter of the set $\{\{1,2\},\{1,2,3\},\{1,2,6\},\{1,2,7\}\}$.

b) Verify completeness of the metric spaces $(\mathbb{N},d_1)$ and $(\mathbb{N},d_2)$ where $d_1(x,y)=|x-y|;\ x,y\in\mathbb{N}$ and $d_2(x,y)=\left|\frac{1}{x}-\frac{1}{y}\right|;\ x,y\in\mathbb{N}$.

c) Which one of the following sets are G_δ in $\mathbb{R}$? Justify your answer. (i) (0,1) (ii) [0,1] and (iii) (0,1].

d) Let A be an uncountable set in $\mathbb{R}$. Prove that A has a subset which is not closed in $\mathbb{R}$.

e) Prove that the union of two bounded sets in a metric space is also bounded. Use it to show that a Cauchy sequence in a metric space is bounded.

f) Describe all Cauchy sequences in $(\mathbb{Q},d_1)$ and $(\mathbb{Q},d_2)$ where d_1, d_2 are defined by $d_1(x,y)=|x-y|;\ x,y\in\mathbb{Q}$ and $d_2(x,y)=\begin{cases}1, x\neq y\\ 0, x=y\end{cases};\ x,y\in\mathbb{Q}$.

g) Prove that the metric space ℓ_∞ is not separable.

2. **Answer any two questions :** [2×5]

a) State and prove Cauchy criterion for uniform convergence of a sequence of function. [1+4]

b) i) If a sequence of continuous function $\{f_n\}$ is uniformly convergent to a function f on $D\subset\mathbb{R}$, prove that f is also continuous on D.

ii) Let $f:\mathbb{R}\to\mathbb{R}$ be uniformly continuous on $\mathbb{R}$. For each natural number n, let $f_n(x)=f\left(x+\frac{1}{n}\right), x\in\mathbb{R}$. Prove that the sequence $\{f_n\}$ in uniformly convergent on $\mathbb{R}$.

c) Prove that $\{f_n\}$ is uniformly convergent on [a,b] to f if and only if $\lim_{n\to\infty}M_n=0$ where $M_n=\sup_{x\in[a,b]}|f_n(x)-f(x)|$. Examine uniform convergence of $\{f_n\}$ on [0,1] where $f_n(x)=nx(1-x)^n$, for all $x\in[0,1]$. [2+3]

d) For each $n\in\mathbb{N}$, $f_n(x)=\frac{x}{1+nx^2}, x\in[0,1]$. Show that $\{f_n\}$ converges uniformly on [0,1]. Show that a sequence of bounded functions convergent uniformly to a bounded function. [2+3]

Group – B

3. **Answer any two questions :** [2×5]

a) Prove that the locus of the extremity of the polar subtangent of the curve $\frac{1}{r}+f(\theta)=0$ is $\frac{1}{r}=f'\left(\frac{\pi}{2}+\theta\right)$.

b) The curve $r=ae^{\theta\cot\alpha}$ cuts any radius vector in the consecutive points P_1, P_2, ...,P_n, P_{n+1}, ... If ρ_n denotes the radius of curvature at P_n, prove that $\frac{1}{m-n}\ln\left(\frac{\rho_m}{\rho_n}\right)$ is constant for all integral values of m and n.

c) Find the asymptotes of the curve : $x^3-x^2y-xy^2+y^3+2x^2-4y^2+2xy+x+y+1=0$.

4. **Answer any three questions :** [3×5]

a) Solve $x\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}+2y=x^3e^x$ after the determination of a solution of its reduced equation.

b) Find the eigen-values and eigen-functions of $\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0$; $y'(1)=0=y'(e^{2\pi})$ where $\lambda>0$.

c) Solve the system of simultaneous equations $\frac{dx}{dt}+4x+3y=t$; $\frac{dy}{dt}+2x+5y=e^t$.

d) Solve : $\frac{dx}{x^2+a^2}=\frac{dy}{xy-az}=\frac{dz}{xz+ay}$.

e) Find *f*(y) such that the total differential $\frac{yz+z}{x}dx-zdy+f(y)dz=0$ is integrable. Hence solve it.

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